Chapter 12
The z and t Tests of Hypotheses

12.3 **a.** Six-step method:

1. H_0: $\mu = 24{,}000$
2. H_a: $\mu < 24{,}000$
3. $\alpha = .05$
4. $\bar{x}_s = 23{,}500$, $z_s = (23500 - 24000)/(\sigma/\sqrt{[n]}) = -500/(4000/10) = -1.25$
5. Rejection region is $z < z_c = -1.6448$ (or $\bar{x} < \$23{,}342.06$). We do not reject H_0 because z_s is not in rejection region (or $\bar{x}_s = 23{,}500$ is not in the rejection region)
6. Although this random sample of 100 35-year-olds in North Dakota had an average income that was \$500 below the national average, there is insufficient evidence to conclude that average income of all 35-year-olds in North Dakota is lower than the national average.

b. $\beta = \Pr(\text{Retain } H_0 \mid \mu = 23{,}600, \sigma = 4000) = \Pr(\bar{X} > 23{,}342.06 \mid \mu = 23{,}600, \sigma = 4000) =$ 0.7405. The power of the test is $1 - \beta = .2595$. The test procedure has about a 25.95% chance of detecting an average income \$400 below the national average if such a difference were present.

12.5 Six-step method:

1. H_0: $\mu = 612$
2. H_a: $\mu > 612$
3. $\alpha = .05$
4. $\bar{x}_s = 630$, $z_s = (630 - 612)/(\sigma/\sqrt{[n]}) = 18/(80/10) = 2.25$
5. Rejection region is $z > z_{0.95} = 1.64485$ or $\bar{x} > 625.16$. $\bar{x}_s$ is in the rejection region.

P-value = $\Pr(Z > z_s) = \Pr(Z > 2.25) = 0.0122 < \alpha$, so reject H_0.

6. There is sufficient evidence (P-value = .0122) to conclude that the mean SAT verbal scores have gone up at this university.

12.7 H_0: $\pi = 0.5$, H_a: $\pi \neq 0.5$, $n = 121$ and $p = 0.64$ so $k_s = 77$ or 78. A conservative estimate of the P-value is to use $k_s = 77$ and $z_s = \dfrac{76.5 - 60.5}{\sqrt{30.25}} = 2\tfrac{10}{11}$.

$$\text{P-value} = 2\Pr(Z \geq 2\tfrac{10}{11}) \approx 0.0036.$$

The P-value is less than the significance level $\alpha = 0.05$, so the null hypothesis is rejected in favor of the alternative hypothesis $\pi \neq 0.5$.

12.9 **a.** Six-step method:

1. H_0: Player has not improved $\pi = 0.55$
2. H_a: Player has improved $\pi > 0.55$
3. $\alpha = .05$
4. $n = 60$, $k_s = 40$ $z_s = (k_s - n\pi)/(\sqrt{[n\ \pi\ (1-\pi)]}) = (39.5 - 33)/(\sqrt{[14.85]}) = 1.6867$.

(Without continuity correction we get $z_s = (k_s - n\pi)/(\sqrt{[n\ \pi\ (1-\pi)]}) = (40 - 33)/(\sqrt{[14.85]}) =$ 1.816)

5. P-value = $Pr(Z > 1.6867) = 0.0458 < \alpha = 0.05$; so reject H_0.

(Without continuity correction we get P-value = $Pr(Z > 1.816) = 0.346$)

6. There is sufficient evidence (P-value = 0.0458) to conclude that the player has improved (at the 5% level of significance).

b. Steps 1 – 4 are the same as in part a.

5. P-value = $Pr(Z > 1.6867) = 0.0458 > \alpha = 0.01$; so retain H_0.
6. There is insufficient evidence (P-value = 0.0458) to conclude that the player has improved (at the 1% level of significance).

c. Rejection region for $\alpha = 0.01$: $z > z_{0.99} = 2.3264$ or $p > .55 + 2.32644\sqrt{[(.55)(.45)/60]} = 0.6994$.
$\alpha = Pr(\text{Retain H0} \mid \pi = 0.70) = Pr(p < 0.6994 \mid \pi = 0.70) = Pr(Z < (0.6994 - 0.70)/\sqrt{[(.70)(.30)/60]}) = 0.4960$

12.11 Six-step method:

1. H_0: $\mu = 140$
2. H_a: $\mu \neq 140$
3. $\alpha = .05$
4. $\bar{x}_s = 137$, $z_s = (137 - 140)/(\sigma/\sqrt{[n]}) = -3/(12/\sqrt{[81]}) = -2.25$
5. Critical values are $z_c = \pm z_{0.975} = \pm 1.96$ or $\bar{x}_c = 140 \pm 1.96(12/\sqrt{[81]}) = 140 \pm 2.6133$. Note that $\bar{x}_s$ is in the rejection region, so reject H_0.

P-value = $2Pr(Z < z_s) = 2Pr(Z < -2.25) = 0.0244 < \alpha$, so reject H_0.

6. There is sufficient evidence (P-value = 0.0244) to conclude that the population mean is not 140.

12.13 Six-step method:

1. H_0: $\mu = 1.5$ inches
2. H_a: $\mu \neq 1.5$ inches
3. $\alpha = .05$
4. $n = 400$, $\bar{x}_s = 1.504$, $z_s = (1.504 - 1.500)/(\sigma/\sqrt{[n]}) = 0.004/(0.075/\sqrt{[400]}) = 1.0667$
5. Critical values are $z_c = \pm z_{0.975} = \pm 1.96$ or $\bar{x}_c = 1.500 \pm 1.96(0.075/\sqrt{[400]}) = 1.500 \pm 0.00735$. Note that $\bar{x}_s$ is not in the rejection region, so retain H_0.

P-value = $Pr(|Z| > |z_s|) = 2Pr(Z > 1.0667) = 0.29 > \alpha$, so retain H_0.

6. There is insufficient evidence (P-value = 0.29) to conclude that the machine needs adjustment.

12.15 Six-step method:

1. H_0: $\mu = 15$
2. H_a: $\mu > 15$
3. $\alpha = .05$
4. $n = 18$, $\bar{x}_s = 16.3$, $t_s = (16.3 - 15)/(s/\sqrt{[n]}) = 1.3/(3.32/\sqrt{[18]}) = 1.6613$
5. Rejection region is $t > t_c = t_{17,0.95} = 1.7396$ or $\bar{x} > \bar{x}_c = 15 + 1.7396(3.32/\sqrt{[18]}) =$ 16.3613. Note that $\bar{x}_s$ is not in the rejection region (or $t_s < 16.3613$); so retain H_0.

Using TI-83, P-value = $\Pr(T_{17} > t_s) = \Pr(T_{17} > 1.6613) = 0.0575 > \alpha$; so retain H_0.
Using Table 4 we obtain $.05 <$ P-value $< .10$; so retain H_0.

6. There is insufficient evidence ($.05 <$ P-value $< .10$) to conclude that the population mean is greater than 15.

12.17 Since the sample size is large, we estimate the population standard deviation σ by the sample standard deviation s and use the z test. The t test will give essentially the same result.

1. H_0: $\mu = \$45,000$
2. H_a: $\mu < \$45,000$
3. $\alpha = .05$
4. $\bar{x}_s = 43000$, $z_s = (43000 - 45000)/(\sigma/\sqrt{[n]}) = -2000/(10000/7) = -1.400$
5. Rejection region is $z < z_c = -1.6448$ (or $\bar{x} < \$42,650.29$). We do not reject H_0 because z_s is not in the rejection region (or $\bar{x}_s = \$43,000$ is not in the rejection region)

P-value = $\Pr(\bar{X} < 43000) = \Pr(Z < -1.400) = 0.0808 > \alpha$; so we do not reject H_0.

6. Although this random sample of 49 starting salaries had a mean that was \$2000 below the claimed mean for the profession, there is insufficient evidence to conclude that mean starting salaries in this profession is below \$45,000.

12.19 Here we use the claimed population standard deviation $\sigma = 2$mg.

1. H_0: $\mu = 30$
2. H_a: $\mu \neq 30$
3. $\alpha = .05$
4. $\bar{x}_s = 725/25 = 29$, $z_s = (29 - 30)/(\sigma/\sqrt{[n]}) = -1/(2/\sqrt{[25]}) = -2.5$
5. P-value = $2\Pr(T < z_s) = 2\Pr(Z < -2.5) = 2(.0062) = .0124 < \alpha$, so reject H_0.
6. There is sufficient evidence (P-value = 0.0124) to conclude that vitamin B-2 content is not 30 mg; the pills have less than 30 mg of vitamin B-2.

12.21 P-value = $\Pr(Z > (3.5 - 2.85)/(.45/\sqrt{[160]})) = \Pr(Z > 18.2708) = 7\cdot10^{-41}$.
The difference cannot be explained by sampling variability. Most likely people were not being truthful.

12.23 Find k_s for which $\Pr(K < k_s) < .07$.
Using the normal approximation we obtain invNorm(210,(210)(.9),√[(210)(.9)(.1)]) = 182.5841.
So if the sample count is 182 or less, then the null hypothesis is rejected.

12.25
1. H_0: $\pi = 102/350$
2. H_a: $\pi < 102/350$
3. $\alpha = .01$
4. $n = 100$, $k_s = 9$, $z_s = (9.5 - n\pi)/(\sqrt{[n\pi(1-\pi)]}) = -19.6429/4.5442 = -4.3226$.

Note the continuity correction.

5. P-value = $\Pr(Z < -4.3226) = 0.000008 < \alpha$, so reject H_0.
6. The small fraction of women chosen cannot be explained by chance variation.

The judge was choosing too few women.

12.27 **a.**

H_0: $\mu = 750, \sigma = 50$
H_a: $\mu > 750, \sigma = 50$

b. Curve is shown in Answers section of book.

Power=

$$1-\beta = \Pr(\overline{X} > 761.63 \text{ hours} \mid \mu) = \Pr\left(Z > \frac{761.63 - \mu}{5}\right) = 1 - \Phi\left(\frac{761.63 - \mu}{5}\right)$$

c. $\beta = \Pr(\text{Retain } H_0 \mid \mu = 750) =$

$$\Pr(\overline{X} \le 761.63 \mid \mu = 750) = \Pr\left(Z \le \frac{761.63 - 750}{5}\right) = \Phi(2.326) \approx 0.98999$$

d. $\Pr(\text{Reject } H_0 \mid \mu = 760) =$

$$\Pr(\overline{X} > 761.63 \mid \mu = 760) = \Pr\left(Z > \frac{761.63 - 760}{5}\right) = 1 - \Phi(0.326) \approx 0.3722$$

$\Pr(\text{Reject } H_0 \mid \mu = 770) =$

$$\Pr(\overline{X} > 761.63 \mid \mu = 770) = \Pr\left(Z > \frac{761.63 - 770}{5}\right) = 1 - \Phi(-1.664) \approx 0.9529$$