Chapter 12
The z and t Tests of Hypotheses

12.3 a. Six-step method:

1. \( H_0: \mu = 24,000 \)
2. \( H_a: \mu < 24,000 \)
3. \( \alpha = 0.05 \)
4. \( \bar{x} = 23,500, z_s = (23500 - 24000)/(\sigma/\sqrt{n}) = -500/(4000/10) = -1.25 \)
5. Rejection region is \( z < z_{c} = -1.6448 \) (or \( \bar{x} < 23,342.06 \)). We do not reject \( H_0 \) because \( z_s \) is not in rejection region (or \( \bar{x} = 23,500 \) is not in the rejection region).
6. Although this random sample of 100 35-year-olds in North Dakota had an average income that was $500 below the national average, there is insufficient evidence to conclude that average income of all 35-year-olds in North Dakota is lower than the national average.

b. \( \beta = Pr(\text{Retain } H_0 \mid \mu = 23,600, \sigma = 4000) = Pr(\bar{X} > 23,342.06 \mid \mu = 23,600, \sigma = 4000) \approx 0.7405. \) The power of the test is \( 1-\beta = 0.2595. \) The test procedure has about a 25.95% chance of detecting an average income $400 below the national average if such a difference were present.

12.5 Six-step method:

1. \( H_0: \mu = 612 \)
2. \( H_a: \mu > 612 \)
3. \( \alpha = 0.05 \)
4. \( \bar{x} = 630, z_s = (630 - 612)/(\sigma/\sqrt{n}) = 18/(80/10) = 2.25 \)
5. Rejection region is \( z > z_{0.95} = 1.64485 \) or \( \bar{x} > 625.16. \) \( \bar{x} \) is in the rejection region.
6. There is sufficient evidence (P-value = .0122) to conclude that the mean SAT verbal scores have gone up at this university.

12.7 \( H_0: \pi = 0.5, H_a: \pi \neq 0.5, n = 121 \) and \( p = 0.64 \) so \( k_s = 77 \) or 78. A conservative estimate of the P-value is to use \( k_s = 77 \) and \( Z_s = \frac{76.5 - 60.5}{\sqrt{30.25}} = \frac{2}{\sqrt{11}}. \)

\[ P\text{-value} = 2Pr(Z \geq \frac{2}{\sqrt{11}}) \approx 0.0036 \]

The P-value is less than the significance level \( \alpha = 0.05 \), so the null hypothesis is rejected in favor of the alternative hypothesis \( \pi \neq 0.5. \)
12.9 a. Six-step method:
1. $H_0$: Player has not improved $\pi = 0.55$
2. $H_a$: Player has improved $\pi > 0.55$
3. $\alpha = .05$
4. $n = 60, k_s = 40, z_s = (k_s - n\pi)/(\sqrt{n}\pi(1 - \pi)) = (39.5 - 33)/(\sqrt{14.85}) = 1.6867.$
   (Without continuity correction we get $z_s = (k_s - n\pi)/(\sqrt{n}\pi(1 - \pi)) = (40 - 33)/(\sqrt{14.85}) = 1.816$)
5. $P$-value = Pr$(Z > 1.6867) = 0.0458 < \alpha = 0.05$; so reject $H_0$.
   (Without continuity correction we get $P$-value = Pr$(Z > 1.816) = 0.346$)
6. There is sufficient evidence ($P$-value = 0.0458) to conclude that the player has improved (at the 5% level of significance).

b. Steps 1 – 4 are the same as in part a.
5. $P$-value = Pr$(Z > 1.6867) = 0.0458 > \alpha = 0.01$; so retain $H_0$.
6. There is insufficient evidence ($P$-value = 0.0458) to conclude that the player has improved (at the 1% level of significance).

c. Rejection region for $\alpha = 0.01$: $z > z_{0.01} = 2.3264$ or $p > .55 + 2.3264\sqrt{(0.55)(0.45)/60} = 0.6994.$
   $\alpha = \Pr(Retain H_0 | \pi = 0.70) = \Pr(p < 0.6994 | \pi = 0.70) = \Pr(Z < (0.6994 - 0.70)/\sqrt{(0.70)(0.30)/60}) = 0.4960$

12.11 Six-step method:
1. $H_0$: $\mu = 140$
2. $H_a$: $\mu \neq 140$
3. $\alpha = .05$
4. $\bar{x}_s = 137, z_s = (137 - 140)/(\sigma/\sqrt{n}) = -3/(12/\sqrt{81}) = -2.25$
5. Critical values are $z_c = \pm z_{0.975} = \pm 1.96$ or $\bar{x}_c = 140 \pm 1.96(12/\sqrt{81}) = 140 \pm 2.6133$. Note that $\bar{x}_s$ is in the rejection region, so reject $H_0$.
   P-value = 2Pr$(Z < z_s) = 2Pr(Z < -2.25) = 0.0244 < \alpha$, so reject $H_0$.
6. There is sufficient evidence ($P$-value = 0.0244) to conclude that the population mean is not 140.

12.13 Six-step method:
1. $H_0$: $\mu = 1.5$ inches
2. $H_a$: $\mu \neq 1.5$ inches
3. $\alpha = .05$
4. $n = 400, \bar{x}_s = 1.504, z_s = (1.504 - 1.500)/(\sigma/\sqrt{n}) = 0.004/(0.075/\sqrt{400}) = 1.0667$
5. Critical values are $z_c = \pm z_{0.975} = \pm 1.96$ or $\bar{x}_c = 1.500 \pm 1.96(0.075/\sqrt{400}) = 1.500 \pm 0.00735$. Note that $\bar{x}_s$ is not in the rejection region, so retain $H_0$.
   P-value = Pr$(Z > 1.0667) = 0.29 > \alpha$, so retain $H_0$.
6. There is insufficient evidence ($P$-value = 0.29) to conclude that the machine needs adjustment.
12.15 Six-step method:
1. \( H_0: \mu = 15 \)
2. \( H_a: \mu > 15 \)
3. \( \alpha = .05 \)
4. \( n = 18, \overline{x} = 16.3, t_c = (16.3 - 15)/(s/\sqrt{n}) = 1.3/(3.32/\sqrt{18}) = 1.6613 \)
5. Rejection region is \( t > t_{17,0.95} = 1.7396 \) or \( x > 15 + 1.7396(3.32/\sqrt{18}) = 16.3613 \). Note that \( \overline{x} \) is not in the rejection region (or \( t \leq 16.3613 \); so retain \( H_0 \).

Using TI-83, \( P \)-value = \( Pr(T_{17} > t_s) = Pr(T_{17} > 1.6613) = 0.0575 > \alpha \); so retain \( H_0 \).

Using Table 4 we obtain \( .05 < P \)-value < .10; so retain \( H_0 \).

6. There is insufficient evidence (.05 < \( P \)-value < .10) to conclude that the population mean is greater than 15.

12.17 Since the sample size is large, we estimate the population standard deviation \( \sigma \) by the sample standard deviation \( s \) and use the \( z \) test. The \( t \) test will give essentially the same result.
1. \( H_0: \mu = $45,000 \)
2. \( H_a: \mu < $45,000 \)
3. \( \alpha = .05 \)
4. \( \overline{x} = 43000, z_s = (43000 - 45000)/(s/\sqrt{n}) = -2000/(10000/7) = -1.400 \)
5. Rejection region is \( z < z_{c} = -1.6448 \) (or \( \overline{x} < $42,650.29 \)). We do not reject \( H_0 \) because \( z_s \) is not in the rejection region (or \( \overline{x} = $43,000 \) is not in the rejection region)

\( P \)-value = \( Pr(\overline{x} < 43000) = Pr(Z < -1.400) = 0.0808 > \alpha \); so we do not reject \( H_0 \).

6. Although this random sample of 49 starting salaries had a mean that was $2000 below the claimed mean for the profession, there is insufficient evidence to conclude that mean starting salaries in this profession is below $45,000.

12.19 Here we use the claimed population standard deviation \( \sigma = 2mg \).
1. \( H_0: \mu = 30 \)
2. \( H_a: \mu \neq 30 \)
3. \( \alpha = .05 \)
4. \( \overline{x} = 725/25 = 29, z_s = (29 - 30)/(s/\sqrt{n}) = -1/(2/\sqrt{25}) = -2.5 \)
5. \( P \)-value = \( 2Pr(T < z_s) = 2Pr(Z < -2.5) = 2(.0062) = .0124 < \alpha \); so reject \( H_0 \).
6. There is sufficient evidence ( \( P \)-value = .0124) to conclude that vitamin B-2 content is not 30 mg; the pills have less than 30 mg of vitamin B-2.

12.21 \( P \)-value = \( Pr(Z > (3.5 - 2.85)/(0.45/\sqrt{160})) = Pr(Z > 18.2708) = 7 \times 10^{-41} \).

The difference cannot be explained by sampling variability. Most likely people were not being truthful.
12.23 Find $k_s$ for which $Pr(K < k_s) < .07$.
Using the normal approximation we obtain
\[
\text{invNorm}(210, (210)(.9), \sqrt{(210)(.9)(.1)}) = 182.5841.
\]
So if the sample count is 182 or less, then the null hypothesis is rejected.

12.25
1. $H_0$: $\pi = 102/350$
2. $H_a$: $\pi < 102/350$
3. $\alpha = .01$
4. $n = 100$, $k_s = 9$, $z_s = (9.5 - n\pi)/(\sqrt{n\pi(1 - \pi)}) = -19.6429/4.5442 = -4.3226$.
Note the continuity correction.
5. P-value = $Pr(Z < -4.3226) = 0.000008 < \alpha$, so reject $H_0$.
6. The small fraction of women chosen cannot be explained by chance variation.
The judge was choosing too few women.

12.27
a. $H_0$: $\mu = 750$, $\sigma = 50$
$H_a$: $\mu > 750$, $\sigma = 50$

b. Curve is shown in Answers section of book.

Power=
\[
1 - \beta = Pr(\bar{X} > 761.63 \text{ hours} | \mu) = Pr\left(Z > \frac{761.63 - \mu}{5}\right) = 1 - \Phi\left(\frac{761.63 - \mu}{5}\right)
\]

c. $\beta = Pr(\text{Retain } H_0 | \mu = 750) =$
\[
Pr(\bar{X} \leq 761.63 | \mu = 750) = Pr\left(Z \leq \frac{761.63 - 750}{5}\right) = \Phi(2.326) \approx 0.98999
\]

d. $Pr(\text{Reject } H_0 | \mu = 760) =$
\[
Pr(\bar{X} > 761.63 | \mu = 760) = Pr\left(Z > \frac{761.63 - 760}{5}\right) = 1 - \Phi(0.326) = 0.3722
\]
\[
Pr(\text{Reject } H_0 | \mu = 770) =$
\[
Pr(\bar{X} > 761.63 | \mu = 770) = Pr\left(Z > \frac{761.63 - 770}{5}\right) = 1 - \Phi(-1.664) = 0.9529
\]