Chapter 9
Waiting Time Random Variables

9.3 $$f(t) = \begin{cases} \frac{1}{4.6} e^{-\frac{1}{4.6}t} & \text{for } t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F(t) = \begin{cases} 1 - e^{-\frac{1}{4.6}t} & \text{for } t \geq 0 \\ 0 & t < 0 \end{cases}$$

9.9 $\lambda = 31/100 = 0.31$/year

a. $f_X(0) = e^{-0.31} \frac{0.31^0}{0!} = 0.7334$

b. $f_X(1) = e^{-0.31} \frac{0.31^1}{1!} = 0.2274$

c. $f_X(2) = e^{-0.31} \frac{0.31^2}{2!} = 0.0352$

d. Pr (Y > 2) = 1 – Pr(Y = 0,1,2) = 1 – {0.7334 + 0.2274 + 0.0352} = 1 – 0.9961 = 0.0039

9.11 $\lambda = 329/365 = 0.9$/day

a. $f_Y(0) = e^{-0.9} \frac{0.9^0}{0!} = 0.4060$

b. $f_Y(1) = e^{-0.9} \frac{0.9^1}{1!} = 0.3660$

c. Pr (Y ≥ 1) = 1 – $f_Y(0)$ = 1 – 0.4060 = 0.5940

9.13 Since λ = 4000(0.003) = 12

Thus $f(12) = e^{-12} \frac{12^{12}}{12!} = 0.1144$

9.15 **a.** Here n = 10, π = 0.50.

$\Pr(5 < Y \le 8) = \Pr(Y = 6,7,8) = ({}_{10}C_6 + {}_{10}C_7 + {}_{10}C_8)(.5)^{10} = 0.3662$

b. Here $\lambda = n\pi = 5$. $\Pr(5 < Y \le 8) = \Pr(Y = 6,7,8) = e^{-5}(\frac{5^6}{6!} + \frac{5^7}{7!} + \frac{5^8}{8!}) = .3159$

c. Here $\mu = n\pi = 5$, $\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{2.5}$. $\Pr(5 < Y \le 8) \xrightarrow[\text{correction}]{\text{continuity}} \Pr(5.5 < Y < 8.5)$

$= P(\frac{5.5-5}{\sqrt{2.5}} < Z < \frac{8.5-5}{\sqrt{2.5}}) = \Phi(2.2136) - \Phi(.3162) = .9866 - .6241 = .3625$

9.17 $f_X(20) = {}_{50}C_{20}(.4)^{20}(.6)^{30} = 0.1146$.

Using the Poisson approximation (λ = 20, n = 20), $f_Y(20) = e^{-20}\frac{20^{20}}{20!} = 0.0888$

9.19 n = 3, π = Pr(H) = .5

Let X = number of heads out of 5 trials; Y = number of heads out of the first 3 trials

Pr (Y = 2 | X = 2) = Pr(Y = 2 and X = 2)/Pr(X = 2) =

$\{{}_3C_2(.5)^2(.5)^1\}\{{}_2C_0(.5)^0(.5)^2\}/\{{}_5C_2(.5)^2(.5)^3\} = 0.3000$

9.23 Here the mean is $\theta = 2$ and the standard deviation is $\theta = 2$

a. Pr (T > 2 +3(2)) = Pr(T > 8) = 1 – $F_T(8) = e^{-(1/2)8} = 0.0183$

b. $\Pr(|T-\mu| > 3\cdot\sigma) \ge \frac{1}{3^2} = 1/9$

9.25 $F(t) = 1 - e^{-\frac{t}{2}}$ $t > 0$

a. Pr(T > 4) = 1 – F(4) = 1 – (1 – e^{-2}) = .135

b. Pr(1 < T < 3) = F(3) – F(1) = (1 – $e^{-1.5}$) – (1 – $e^{-0.5}$) = .3833

c. Here π = Pr(T > 4) = .135 and 1 – π = . 865. Pr(At least one out of 5 will last over 4 years) = 1 – Pr(None will last over 4 years) = 1 – ${}_5C_0(.135)^0(.865)^5$ = 1 – $(.865)^5$ = 0.5157

9.27 **a.** Here $\lambda = 3$. Pr(Y = 0) = $e^{-3}\frac{3^0}{0!}$ = 0.0498

b. Pr(Y = 3) = $e^{-3}\frac{3^3}{3!}$ = 0.2240

c. Pr(Y≥ 2) = 1 – Pr(Y = 0,1) = 1 – 0.1991 = 0.8009

9.29 **a.** $\lambda = 200(0.01) = 2$; $f_Y(1) = e^{-2}\dfrac{2^1}{1!} = 0.2707$

b. $f_Y(2) = e^{-2}\dfrac{2^2}{2!} = 0.2707$

c. $\Pr(Y > 1) = 1 - \Pr(Y = 0,1) = 1 - e^{-2}\left\{\dfrac{2^0}{0!} + \dfrac{2^1}{1!}\right\} = 1 - 0.4060 = 0.5940$

9.31 $\lambda = 36(1/36) = 1$

a. Poisson Approximation	b. Exact Chance
$f_Y(0) = e^{-1}\dfrac{1^0}{0!} = 0.3679$	$f_X(0) = 0.3627$
$f_Y(1) = e^{-1}\dfrac{1^1}{1!} = 0.3679$	$f_X(1) = 0.3731$
$f_Y(2) = e^{-1}\dfrac{1^2}{2!} = 0.1839$	$f_X(2) = 0.1865$
$f_Y(3) = e^{-1}\dfrac{1^3}{3!} = 0.0613$	$f_X(3) = 0.0604$
$f_Y(4) = e^{-1}\dfrac{1^4}{4!} = 0.0153$	$f_X(4) = 0.0142$

9.33 $\lambda = (1/\theta)(30) = (1/43.3)(30) = 0.69.$ $f_Y(2) = e^{-0.69}\dfrac{0.69^2}{2!} = 0.1200$

9.35 $\lambda = 5$. $\Pr(Y > 8) = 1 - \Pr(Y \leq 8) = 1 - \sum_{y=0}^{8} e^{-5}\dfrac{5^y}{y!} = 1 - e^{-5}\left(\dfrac{5^0}{0!} + \cdots + \dfrac{5^8}{8!}\right) = 1 - 0.9319$ $= 0.0681$

9.37 $\lambda = 10$. $\Pr(Y < 5) = \Pr(Y \leq 4) = 0.0293$

9.39 $\lambda = 500/365 = 1.37$ births/day. $\Pr(Y > 4) = 1 - \Pr(Y \leq 4) = 1 - 0.9869 = 0.0131$

9.41 **a.** Let T be the time to failure with mean θ and density function $f_T(t) = (1/\theta)e^{-t/\theta}$. Since $\Pr(T \le 1) = 0.10 = F_T(1) = 1 - e^{-1/\theta}$, solving for θ, we have $\theta = \dfrac{-1}{\ln(1-0.10)} = 9.4912$ years.

b. $\Pr(1 < T \le 2) = F_T(2) - F_T(1) = (1 - e^{-2/\theta}) - (1 - e^{-1/\theta}) = e^{-1/\theta} - e^{-2/\theta} = .9 - (.9)^2 = 0.09$

9.43 **a.** $\bar{y} = \dfrac{1}{n}\sum y \cdot f = \dfrac{10086}{2608} = 3.8673$ from the table below.

b. Expected frequencies = 2608·Pr(Y = y) as shown in table below.

y	f	y·f	Expected
0	57	0	54.5457
1	203	203	210.9446
2	383	766	407.893
3	525	1575	525.8149
4	532	2128	508.371
5	408	2040	393.2046
6	273	1638	253.44
7	139	973	140.0184
8	45	360	67.68663
9	27	243	29.08495
10	16	160	11.24802
Total	2608	10086	2602.252

9.45 Let λ denote the rate items are confiscated per hour. Let A denote the number confiscated at Terminal A, and let B denote the number confiscated at Terminal B, and let S = A + B. Then A, B, and S are all Poisson random variables with means 3λ, 5λ, and 8λ, respectively. The random variables A and B are independent.

If the inspector confiscates 6 items in both terminals, the conditional probability that 0 were confiscated in Terminal A (and hence 6 were confiscated in Terminal B) is

Pr(A = 0 | S = 6) = Pr (A = 0 and B = 6 | S = 6) = [Pr (A = 0)·Pr (B = 6)]/Pr (S = 6) =

$$[e^{-3\lambda}\frac{(3\lambda)^0}{0!}\cdot e^{-5\lambda}\frac{(5\lambda)^6}{6!}]/(e^{-8\lambda}\frac{(8\lambda)^6}{6!}) = (5/8)^6 = 0.05960$$

9.47 Note $x_q = -ln(q)$. Also $Q'(x_q) = -Q(x_q) = q$. Thus, for $0<q<1$, we have

$$f_Q(q) = \frac{-f_X(x_q)}{Q'(x_q)} = \frac{-e^{-x_q}}{q} = \frac{q}{q} = 1$$

That is, Q is a uniform random variable on the unit interval.