Chapter 8
Normal Random Variables

8.1 **a.** $x = 142$, $\mu = 135$, $\sigma = 5$, $z = (142 - 135)/5 = 7/5 = 1.4$

b. $x = 135$, $\mu = 135$, $\sigma = 5$, $z = (135 - 135)/5 = 0.0$

c. $x = 102$, $\mu = 135$, $\sigma = 5$, $z = (102 - 135)/5 = -32/5 = -6.4$

d. $x = 130$, $\mu = 135$, $\sigma = 5$, $z = (130 - 135)/5 = -5/5 = -1.0$

8.3 **a.** $z = -1$, so x is 1 σ's below the mean μ or $x = -50 - 1(6) = -56$

b. $z = 0$, so x is the mean or $x = -50$

c. $z = 1.2$, so x is 1.2 σ's above the mean or $x = -50 + 1.2(6) = -42.8$

d. $z = -2.5$, so x is –2.5 σ's above the mean or $x = -50 - 2.5(6) = -65$

e. $z = 4.5$, so x is 3 σ's above the mean or $x = -50 + 4.5(6) = -23$

8.5 **a.** Use TI-83, normalcdf(0,1.25) = 0.3944.
Or Table 3, $\Phi(1.25) - \Phi(0) = 0.89435 - 0.5 = 0.39435$

b. Use TI-83, normalcdf(0.25,1.25) = 0.2956.
Or Table 3, $\Phi(1.25) - \Phi(0.25) = 0.89435 - 0.59871 = 0.2956$

c. Same as part b because of symmetry of normal curve: 0.2956

d. Same as part a because of symmetry of normal curve: 0.3944

8.7 **a.** normalcdf(–99999999,60,52,8) = 0.8413.
Or $\Phi((60 - 52)/8) = \Phi(1.00) = 0.8413$

b. normalcdf(52,60,52,8) = 0.3413.
Or Answer to part a minus 0.50: $0.8413 - 0.50 = 0.3413$.

c. normalcdf(44,52,52,8) = 0.3413.
Same as part b by symmetry of normal curve.

d. normalcdf(46,60,52,8) = 0.6147.
Or $\Phi((60 - 52)/8) - \Phi((46 - 52/8) = 0.8413 - 0.2266 = 0.6147$

e. normalcdf(60,62,52,8) = 0.0530.
Or $\Phi((62-52)/8) - \Phi((60-52)/8) = 0.8944 - 0.8413 = 0.0531$

f. normalcdf(62,99999999,52,8) = 0.1056.
Or $1 - \Phi((62-52)/8) = 1 - 0.8944 = 0.1056$

8.9 E(Z) = 0,

$$\mathrm{E}(\mathrm{Z}^2) = \mathrm{Var}(\mathrm{Z}) + [\mathrm{E}(\mathrm{Z})]^2 = 1 + 0 = 1$$

$$E(Z^3) = \int_{-\infty}^{0} z^3 \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\,dz + \int_{0}^{\infty} z^3 \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\,dz\,.$$

Make the change of variables $y = -z$ in the first integral to show it is the opposite of the second.

$$\int_{-\infty}^{0} z^3 \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\,dz = \int_{\infty}^{0} y^3 \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}}\,dz = -\int_{0}^{\infty} z^3 \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\,dz\,.$$

Then make the change of variables $x = z^2/2$ in the second integral to show

$$\int_{0}^{\infty} z^3 \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}}\,dz = 2\int_{0}^{\infty} xe^{-x}\,dx = 2 < \infty \text{ Thus } E(Z^3) = 0$$

8.11 Q_1 = invNorm(0.25,145,30) = 124.765.
Or use Table 3 and interpolate to obtain $\Phi(-0.6745) \approx 0.25$.
Then height is 0.6745σ's below the mean: x = 145 – 0.6745(30) = 124.765.

Q_3 = invNorm(0.75,145,30) = 165.235.
Or use Table 3 and interpolate to obtain $\Phi(0.6745) \approx 0.75$.
Then height is 0.6745σ's above the mean: x = 145 + 0.6745(30) = 165.235.

IQR = $\mathrm{Q}_3 - \mathrm{Q}_1$ = 165.235 – 124.765 = 40.47

8.13 **a.** normalcdf(–9999999,166,176,30) = 0.3694.
Or $\Phi((166 - 176)/30) = \Phi(-0.0333) = 0.3694$

b. normalcdf(166,9999999,176,30) = 0.6306.
Or 1 – (answer to part a) = 1 – 0.3694 = 0.6306.

c. normalcdf(–9999999,186,176,30) = 0.6306.
Or same as part b by symmetry of normal curve.

d. normalcdf(220,9999999,176,30) = 0.0712.
Or $1 - \Phi((220,176)/30) = 1 - 0.9288 = 0.1712$.

e. normalcdf(260,9999999,176,30) = 0.0026.
Or $1 - \Phi((260,176)/30) = 1 - 0.9974 = 0.0026$.

8.15 $z_{0.25} = -0.6745$, so $x_{0.25} = 167 + (-0.6745)(30) = 146.8$
$z_{0.90} = +1.2815$, so $x_{0.90} = 167 + (1.2815)(30) = 205.4$
$z_{0.95} = +1.6449$, so $x_{0.25} = 167 + (1.6449)(30) = 216.3$

8.17 $1 - \Pr(64 < X < 78) = 1 - \Pr(-2 < Z < 3.6) = 1 - 0.9771 = 0.0229 = 2.29\%$

8.19 Let X be the ACT test scores and let Y be the SAT test scores.
$\Pr(Y < 700) = \Pr(Z < (700 - 500)/100) = \Pr(Z < 2) = 0.9773$
$\Pr(X < 24) = \Pr(Z < (24 - 18)/6) = \Pr(Z < 1) = 0.8413$
Aras is in the 97th percentile and Rimas is in the 84th percentile.
Thus, Aras did better.

8.21 **a.** $\Pr(IQ = 101) = \Pr(100.5 < IQ < 101.5) \approx \Phi(\frac{101.5-100}{15}) - \Phi(\frac{100.5-100}{15}) = 0.0265$

b. $\Pr(IQ > 130) = \Pr(IQ > 130.5) \approx 1 - \Phi(\frac{130.5-100}{15}) = 1 - \Phi(2.0333) = 0.0210$

c. $\Pr(IQ \geq 130) = \Pr(IQ > 129.5) \approx 1 - \Phi(\frac{129.5-100}{15}) = 1 - \Phi(1.9667) = 0.0246$

d. $\Pr(98 \leq IQ \leq 102) = \Pr(97.5 < IQ < 102.5) \approx \Phi(\frac{102.5-100}{15}) - \Phi(\frac{97.5-100}{15}) = 0.1324$

8.23 Pr(K = 50) = Pr(49.5 < K < 50.5) $\approx \Phi(\frac{50.5-50}{.5}) - \Phi(\frac{50.5-50}{.5}) = \Phi(0.1) - \Phi(-0.1) =$ 0.07966

8.25 The event is $\{8 \le K \le 10\} = \{8,9,10\}$. The opposite event is $\{0,1,\ldots,7\} \cup \{11,12,\ldots,15\}$.
Go halfway: the continuity correction is
Pr$\{8 \le K \le 10\}$ =(c.c.) Pr(7.5 < K < 10.5). $\mu_K = n\pi = 7.5$, $\sigma_K = \sqrt{n\pi(1-\pi)} = 1.93649$.
Pr(7.5 < K < 10.5) =(n.a.) Pr((7.5 – 7.5)/1.93649 < Z < (10.5 – 7.5)/1.93649) =
$\Phi(1.291) - 0.5000 = 0.4016$.

Compare this with the exact probability:
binomcdf(15,0.50,10) – binomcdf(15,0.50,7) = 0.9408 – 0.5000 = 0.4408.

8.27 **a.** Pr(Boys $\ge$ 54) = Pr(Boys > 53.5) $\approx 1 - \Phi(\frac{53.5-50}{.5}) = 1 - \Phi(.7) = 0.24196$

b. Pr(Boys > 54) = Pr(Boys > 54.5) $\approx 1 - \Phi(\frac{54.5-50}{.5}) = 1 - \Phi(.9) = 0.18406$

8.29 **a.** Pr(K $\le$ 11) = (c.c.) Pr(K < 11.5) $\approx$ (n.a.) Pr(Z < (11.5 – (40)(0.395))/$\sqrt{(40)(.395)(.605)}$)
= Pr(Z < –1.39) = 0.0821

b. Pr(K $\le$ 11) = 1 – binomcdf(40,0.395,11) = 0.0800

c. The normal approximation is recommended: $n\pi = 15.8$ and $n(1-\pi) = 24.2$ are at least 5.

8.31 invNorm(0.05) = –1.645, so $IQ_{.05}$ = 100 – 1.645(15) = 75.3.
invNorm(0.25) = –0.6745, so $IQ_{.25}$ = 100 – 0.6745(15) = 89.9.
invNorm(0.75) = 0.6745, so $IQ_{.75}$ = 100 + 0.6745(15) = 110.1.
invNorm(0.95) = 1.645, so $IQ_{.95}$ = 100 + 1.645(15) = 124.7.

8.33 **a.** Pr(In excess of 3.3 minutes) = Pr(X > 3.3) = 1 – Pr($X \le 3.3$) = 1 – $\Phi(1.5)$ = 1 – 0.9332 = 0.0668

b. Pr(2.9 < X < 3.1) = $\Phi(0.5) - \Phi(-0.5)$ = 0.3829

c. invNorm(0.01) = –2.32635. So x = 3 – 2.32635(0.2) = 2.5347

8.35 **a.** 99^{th} percentile score is invNorm(0.99,100,15) = 135

b. Pr(IQ > 120) = (c.c.) Pr(IQ > 120.5) = normalcdf(120.5,99999999,100,15) = 0.0859

c. Pr(All have IQ over 120) = $(0.0859)^4$ = 0.00005

d. Pr(At least one has IQ over 120) = 1 – Pr(Nobody has IQ over 120) = 1 – $(1 – 0.00859)^4$ = 0.3017

e. Pr(Three of the four will have IQ over 120) = $\binom{4}{3}(.00859)^3(1-.00859)^1$ = 0.0023

8.37 **a.** Here μ = 16.15, σ = 0.05 ounces. Pr(X > 16.2) = 0.1587

b. Here μ = 16.05, σ = 0.05 ounces. Pr(X < 16) = 0.1587

c. The 5th percentile is z = invNorm(0.05) = –1.645. Solve (11.9 – μ)/0.1 = –1.645.
Solution: the machine should be set so that μ is at least 16.0822 ounces.

8.39 $\Pr(X > 60 \mid \mu = 45, \sigma = 10 = \Pr\left(Z > \frac{60-45}{10}\right) \approx 1 - \Phi(1.5) \approx 6.68\%$

8.41 $\mu = np = 1000(0.15) = 150,\ \sigma = \sqrt{np(1-p)} = \sqrt{1000(.15)(.85)} = 11.29$

a. Pr(K <120) =(c.c) Pr(K < 119.5) $\approx \Phi(\frac{119.5-150}{11.29})$ = 0.0035

b. z = invNorm(0.90) = 1.282 so k = 150 + 1.282(11.29) = 164.5 claims

8.43 Let K be the number who voted for the incumbent. Since n = 120, we have
$\mu = n\pi = 120(.56) = 67.2$ and $\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{(120)(.56)(.44)} = 5.44$.
Pr(K < 60) = (c.c.) Pr(K < 59.5) ≈ (n.a.) Pr(Z < (59.5 – 67.2)/5.44) = $\Phi(-1.416)$ = 0.0784.
So there is a 7.838% probability that the exit poll incorrectly predicts the winner.

8.45 Let K be the number of aces in 120 rolls of a die.
n = 120, $\mu = n\pi = 120(1/6) = 20$, and $\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{(120)(1/6)(5/6)}$] = 4.08.
Pr(K = 24) = (c.c.) Pr(23.5 < K < 24.5) ≈ (n.a.) Pr((23.5 – 20)/4.08 < Z < (24.5 – 20)/4.08) =
Pr(0.857 < Z <1.102) = $\Phi(1.102) - \Phi(.857) = 0.0605$

8.47 Let X be the length of this section of the wall.

a. E(X) = 12 inches/brick * 100 bricks = 1200 inches = 100 feet

b. SD(X) = $\sqrt{100}.25$ = 2.5 inches

c. Pr(X > 1200 + 5 inches) = P (Z > 5/2.5) = 1 – $\Phi(2)$ = 0.0228

d. No. By the Central Limit Theorem, for large samples, the sum of the lengths of the bricks follows the normal curve, even if the individual lengths do not.

8.49 Let K be the number of people who voted Yes. n = 900, $\mu = n\pi = 900 * .56 = 504$, and
$\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{(900)(.56)(.44)} = 14.89$. Pr(K ≥ 573) = (c.c.) Pr(K > 572.5) = (n.a.)
1 – Φ((572.5 – 504)/14.89) = 0.00000212. A lot of the people were not telling the truth.