Chapter 7
Continuous Random Variables

7.1 f(x) = x/2 for 0 < x < 5; f(x) = 0 elsewhere

7.3 **a.** F(x) = 0 for x < 0; F(x) = (1/2)($3x^2 - x^3$) for 0 < x < 1; F(x) = 1 for x > 1.

b. F(0.25) = 11/128 = 0.0859

c. Pr(X $\geq$ 0.25) = 1 – F(0.25) = 1 – 11/128 = 0.9141

d. F(0.75) = 81/128 = .6328

e. Pr(0.25 < X $\leq$ 0.75) = F(0.75) – F(0.25) = 81/128 – 11/128 = 0.5469.

7.5 **a.**
F(x) = 0 for x < –1,
F(x) = $x^2/2$ + x + 1/2 for –1 < x < 0,
F(x) = $-x^2/2$ + x + 1/2 for 0 < x < 1,
F(x) = 1 for x > 1.

b. F(–0.5) = 1/8

c. P(X $\geq$ 0) = 1 – F(0) = 0.5,

d. F(0.5) = 7/8,

e. Pr(–0.5 < X $\leq$ 0.5) = F(0.5) – F(–0.5) = 7/8 – 1/8 = 6/8

7.7 **a.** Graph not shown:
F(x) = 0 for x < 0;
F(x) = x^2 for 0 $\leq$ x < 1;
F(x) = 1 for x $\geq$ 1

b. F(0.25) = $\int_{-\infty}^{0.25} f(x)dx = \int_{-\infty}^{0} 0dx + \int_{0}^{0.25} 2xdx = 0 + 1/16 = 1/16$

c. Pr(X $\geq$ 0.25) = 1 – F(0.25) = 1 – 1/16 = 15/16

d. F(0.75) = $\int_{-\infty}^{0.75} f(x)dx = \int_{-\infty}^{0} 0dx + \int_{0}^{0.75} 2xdx = 0 + 9/16 = 9/16$

e. Pr(0.25 < X $\leq$ 0.75) = F(0.75) – F(0.25) = 9/16 – 1/16 = 1/2

7.9 For 25th percentile, solve F(x) = 0.25:

$x^2/4 = 0.25$ for $0 \le x < 2$.

Solution: 25th percentile = $x_{0.25} = 1$.

For median (50th percentile), solve F(x) = 0.50:

$x^2/4 = 0.50$ for $0 \le x < 2$

Solution: median = $x_{0.50} = \sqrt{2}$

7.11 To find Q_3, solve $F(x) = 1 - e^{-t/80} = 0.75$. Solution: $Q_3 = 80(\ln 4)$.
To find Q_1, solve $F(x) = 1 - e^{-t/80} = 0.25$. Solution: $Q_1 = 80(\ln 4/3)$.
$IQR = Q_3 - Q_1 = 80(\ln 4) - 80(\ln 4/3) = 80(\ln 3) = 87.8889$

7.13 For the median, solve $F(y) = 1 - (2 - y/3)^2 = 0.50$. Solution median = 3.8787

7.15 mode = 0, (this is point x where f attains its maximum); median = 0, (this is the 50th percentile. By symmetry, it's clear that this occurs at x = 0); IQR = $2 - \sqrt{2}$ = 0.5858

7.17

$$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{0} xf(x)dx + \int_{0}^{+1} xf(x)dx + \int_{+1}^{\infty} xf(x)dx$$

$$= \int_{-\infty}^{0} x(0)dx + \frac{3}{2}\int_{0}^{+1} 2x^2 - x^3 dx + \int_{+1}^{\infty} x(0)dx = 0 + \frac{3}{2}\left(\frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 + 0 = \frac{5}{8}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-\infty}^{0} x^2(0)dx + \frac{3}{2}\int_{0}^{+1} 2x^3 - x^4 dx + \int_{+1}^{\infty} x^2(0)dx$$

$$= 0 + \frac{3}{2}\left(\frac{2x^4}{4} - \frac{x^5}{5}\right)\Bigg|_0^1 + 0 = \frac{9}{20}$$

$$\sigma^2 = E(X^2) - \mu^2 = 9/20 - 25/64 = 19/320$$

$$\sigma = \sqrt{19/320} = 0.2437$$

7.19 $f(x) = F'(x) = x/2$ for $0 < x < 2$.

$$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{0} xf(x)dx + \int_{0}^{+2} xf(x)dx + \int_{+2}^{\infty} xf(x)dx$$

$$= \int_{-\infty}^{0} x(0)dx + \int_{0}^{+2} x\frac{x}{2}dx + \int_{+2}^{\infty} x(0)dx = 0 + \frac{x^3}{6}\Bigg|_0^2 + 0 = \frac{8}{6}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-\infty}^{0} x^2(0)dx + \int_{0}^{2} \frac{x^3}{2} dx + \int_{+1}^{\infty} x^2(0)dx = 2$$

$$\sigma^2 = E(X^2) - \mu^2 = 2 - 16/9 = 2/9 \qquad \sigma = \sqrt{2/9} = 0.4714$$

7.21 f(x) = F'(x) = 1/3 for 3 < x < 6, and f(x) = 0 elsewhere.

$$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{5} x(0)dx + \int_{3}^{6} x(1/3)dx + \int_{6}^{\infty} x(0)dx = \left.\frac{x^2}{6}\right|_3^6 = \frac{36}{6} - \frac{9}{6} = 4.5$$

$$\text{E}(\text{X}^2) = \int_3^6 x^2/3dx = 21$$

$$\sigma^2 = E(X^2) - \mu^2 = 21 - (9/2)^2 = 3/4$$

$$\sigma = \sqrt{3/4} = 0.8660$$

7.23 $$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{0} x(0)dx + \int_{0}^{1} x(2x)dx + \int_{1}^{\infty} x(0)dx = \left.\frac{2x^3}{3}\right|_0^1 = \frac{2}{3}$$

$$\text{E}(\text{X}^2) = \int_3^6 x^2(2x)dx = \frac{1}{2}$$

$$\sigma^2 = E(X^2) - \mu^2 = 2/3 - (1/2)^2 = 5/12 \qquad \sigma = \sqrt{5/12} = 0.6455$$

7.25 $\int_0^1 (x - 2/3)^3 (2x)dx = -1/135$.

The coefficient of skewness is $\dfrac{E(X-\mu)^3}{\sigma^3} = \dfrac{-1/135}{\left(\sqrt{5/12}\right)^3} = 0.0275$

7.27 **a.** Common rule: Approximately 68% of IQ scores are between 85 and 115.
Chebyshev's rule: More than 0% of IQ scores are between 85 and 115.

b. Common rule: Approximately 95% of IQ scores are between 70 and 130.
Chebyshev's rule: More than 75% of IQ scores are between 70 and 130

c. Common rule: Approximately 99.7% (almost all) of IQ scores are between 55 and 145.
Chebyschev's rule: More than 8/9 = 88.89% of IQ scores are between 55 and 145.

7.29 **a.** Graph shown in Answers section of book.

$F(x) = 0$ for $x < 1$;
$F(x) = (x^2 - 1)/48$ for $1 < x < 7$;
$F(x) = 1$ for $x > 7$

b. $Pr(1 < X \le 5) = F(5) - F(1) = 24/48 - 0/48 = 1/2$

c. $E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{1} 0dx + \int_{1}^{7} \frac{x^2}{24}dx + \int_{7}^{\infty} 0dx = 0 + \left.\frac{x^3}{72}\right|_1^7 + 0 = \frac{342}{72} = \frac{19}{4}$

d. $Var(X) = E(X^2) - E^2(X) = \int_{1}^{7} \frac{x^3}{24}dx - \left(\frac{19}{4}\right)^2 = 25 - \left(\frac{19}{4}\right)^2 = \frac{39}{16}$

e. median: Solve $F(x) = (x^2 - 1)/48 = 0.50$. Solution: median = 5

f. mode = 7. This is where f(x) attains its maximum.

7.31 **a.** $F(x) = x^3/8$ for $0 < x < 2$;
$F(x) = 0$ for $x < 0$;
$F(x) = 1$ for $x > 2$.

b. $Pr(X > 1) = 1 - F(1) = 1 - 1/8 = 7/8$

c. $\mu = E(X) = \frac{3}{8}\int_{0}^{2} x^3 dx = \frac{3}{2}$

d. $E(X^2) = 3/8\int_{0}^{1} x^4 dx = \frac{12}{5}$, $SD(X) = \sqrt{E(X^2) - \mu^2} = \sqrt{\frac{12}{5} - \frac{9}{4}} = \sqrt{\frac{3}{20}} = 0.3873$

e. $Pr(\mu - 2\sigma < X < \mu + 2\sigma) = F(\mu + 2\sigma) - F(\mu - 2\sigma) = F(2.2746) - F(.7254) = 1 - 0.1477 = 0.9523$
Caution: $F(2.2746) = 1$ because $F(x) = 1$ for $x > 2$.

7.35 **a.** $\mu = \int_{-1}^{0} x(x+1)\,dx + \int_{0}^{1} x(-x+1)\,dx = -1/6 + 1/6 = 0$

b. $E(X^2) = \int_{-1}^{0} x^2(x+1)dx + \int_{0}^{1} x^2(-x+1)dx = 1/12 + 1/12 = 1/6$

c. $\sigma = \sqrt{E(X^2) - \mu^2} = \sqrt{\frac{1}{6} - 0} = 0.4082$

7.37 **a.** Follow the solution for Example 7.2. Let X_1 and X_2 be the two randomly chosen numbers between 0 and 1. This means that X_1 and X_2 have the same density function f(x) = 1 (for 0 < x < 1), and the same cumulative distribution function F(x) = x (for 0 < x < 1). Let Y be the larger of the two. So if Y is less than some number y, then both X_1 and X_2 must be less than y. Thus F(y) = Pr(Y ≤ y) = Pr(X_1 ≤ y and X_2 ≤ y) = Pr(X_1 ≤ y)Pr(X_2 ≤ y) = y^2, for 0 < y < 1. Also F(y) = 0 for y ≤ 0 and F(y) = 1 for y ≥ 1.

b. f(y) = F′(y) = 0 for y < 0, f(y) = = F′(y) = 2y for 0 < y < 1, f(y) = F′(y) = 0 for y > 1.

c. Pr(0,25 < Y < 0.75) = F(0.75) – F(0.25) = $(0.75)^2 - (0.25)^2$ = 9/16 – 1/16 = 0.50

d. $E(Y) = \int_{-\infty}^{\infty} yf(y)dy = \int_{-\infty}^{0} 0dy + \int_{0}^{1} ydy + \int_{1}^{\infty} 0dy = 0 + \left.\frac{y^2}{2}\right|_0^1 + 0 = \frac{1}{2}$

e. $Var(Y) = E(Y^2) - E^2(Y) = \int_0^1 y^2 dy - \left(\frac{1}{2}\right)^2 = \left(\frac{1}{3}\right) - \left(\frac{1}{4}\right) = \frac{1}{12}$

7.39 **a.** $\mu = \int_{-\infty}^{0} (0)dt + \int_{0}^{\infty} .2te^{-.2t}dt = -te^{-.2t}\Big|_0^{\infty} + \int_0^{\infty} e^{-.2t}dt = 0 + 5 = 5$.

Note integration by parts above: $\int_0^{\infty} udv = (uv - \int vdu)\Big|_0^{\infty}$

b. $\sigma^2 = E(T^2) - \mu^2 = 50 - (5)^2 = 25$, $\sigma = \sqrt{25} = 5$

c. Highest point of the graph of f occurs at t = 0. Mode = 0

d. To find the median (50th percentile), solve $F(t) = 1 - e^{-.2t} = .5$.
Solution: median = 5ln{.5} = 3.4657

7.41 **a.** Pr(T > 1000) = 1 – F(1000) = e^{-1} = 0.3679

b. f(t) = 0 if t < 0; f(t) = $e^{-t/1000}/1000$ if 0 < t

c. $\mu = \int_{-\infty}^{0} t(0)dt + \int_0^{\infty} (.001)te^{-.t/1000}dt = -te^{-t/1000}\Big|_0^{\infty} + \int_0^{\infty} e^{-.t/1000}dt = 0 + 1000 = 1000$

7.43 **a.** $\Pr(X \geq 1) = \int_{1}^{\infty} f(x)dx = \int_{1}^{6} \frac{36 - x^2}{288} dx + \int_{6}^{\infty} 0 dx = \frac{1}{288}(36x - \frac{x^3}{3})\Big|_{1}^{6} = \frac{325}{864}$

b. $\Pr(-3 < X < -1) = \int_{-3}^{-1} \frac{36 - x^2}{288} dx = \frac{1}{288}(36x - \frac{x^3}{3})\Big|_{-3}^{-1} = \frac{190}{864}$

c. $\Pr(X = 0) = \int_{0}^{0} \frac{36 - x^2}{288} dx = 0$. This is an example of the continuity paradox.

d. $E(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-6}^{6} x\frac{36 - x^2}{288} dx = \frac{1}{288}(18x^2 - \frac{x^4}{4})\Big|_{-6}^{6} = 0$,

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-6}^{6} x^2 \frac{36 - x^2}{288} dx = \frac{1}{288}(12x^3 - \frac{x^5}{5})\Big|_{-6}^{6} = \frac{36}{5} = 7.2$

$\sigma = \sqrt{E(X^2) - \mu^2} = \sqrt{7.2 - 0} = 2.6833$

7.45 $E(F) = E(\frac{9}{5}C + 32) = \frac{9}{5}E(C) + 32 = \frac{9}{5}(14) + 32 = 57.2$

$\sigma^2 = Var(F) = Var(\frac{9}{5}C + 32) = \left(\frac{9}{5}\right)^2 Var(C) = \left(\frac{9}{5}\right)^2 (50) = 162$