Chapter 5
Random Variables for Success/Failure Experiments

5.1 \[ \mu = \pi = 1/13 = 0.0769, \]
\[ \sigma = \sqrt{\pi(1 - \pi)} = \sqrt{(1/13)(12/13)} = 0.2665 \]

5.3 \[ \mu = \pi = 6022/8023 = 0.7506, \]
\[ \sigma = \sqrt{\pi(1 - \pi)} = \sqrt{(6022/8023)(2001/8023)} = 0.4327 \]

5.9 In general \( \mu = n\pi \) and \( \sigma = \sqrt{n\pi(1 - \pi)} \)

a. For \( n = 100 \) and \( \pi = 1/3 \), \( \mu = n\pi = 100 \times 1/3 = 33.3333 \) and
\[ \sigma = \sqrt{n\pi(1 - \pi)} = \sqrt{100(1/3)(2/3)} = \sqrt{200 / 3} = 10\sqrt{2 / 3} = 4.7140 \]

b. For \( n = 900 \) and \( \pi = 1/3 \), \( \mu = n\pi = 900 \times 1/3 = 300 \) and
\[ \sigma = \sqrt{n\pi(1 - \pi)} = \sqrt{900(1/3)(2/3)} = \sqrt{1800 / 3} = 30\sqrt{2 / 3} = 14.1421 \]

c. For \( n = 100 \) and \( \pi = 2/3 \), \( \mu = n\pi = 200 \times 2/3 = 66.6667 \) and
\[ \sigma = \sqrt{n\pi(1 - \pi)} = \sqrt{100(1/3)(2/3)} = \sqrt{200 / 3} = 10\sqrt{2 / 3} = 4.7140 \]

d. For \( n = 900 \) and \( \pi = 2/3 \), \( \mu = n\pi = 1800 \times 2/3 = 600 \) and
\[ \sigma = \sqrt{n\pi(1 - \pi)} = \sqrt{900(1/3)(2/3)} = \sqrt{1800 / 3} = 30\sqrt{2 / 3} = 14.1421 \]

5.11 \[ Pr(K = k) = \binom{n}{k}(0.1)^k(0.9)^{10-k} \]

a. \( Pr(K = 0) = \binom{10}{0}(0.1)^0(0.9)^{10} = (1)(1)(0.9)^{10} = 0.3487 \)

b. \( Pr(K = 1) = \binom{10}{1}(0.1)^1(0.9)^9 = (10)(0.1)(0.9)^9 = 0.3874 \)

c. \( Pr(K = 2) = \binom{10}{2}(0.1)^2(0.9)^8 = (45)(0.1)^2(0.9)^8 = 0.1937 \)

d. \( Pr(K < 2) = Pr(K = 0) + Pr(K = 1) = 0.3487 + 0.3874 = 0.7361 \)

e. \( Pr(K \geq 2) = 1 - Pr(K < 2) = 1 - 0.7361 = 0.2639 \)

f. \( Pr(K > 2) = 1 - Pr(K \leq 2) = 1 - (Pr(K=0) + Pr(K = 1) + Pr(K = 2)) = 1 - 0.9298 = 0.0702 \)
5.13  
\(a. \ Pr(K = 1) = 100 \binom{1}{.01}^1 (0.99)^{99} = .3697\)
\(b. \ Pr(K = 0) = 100 \binom{0}{.01}^0 (0.99)^{100} = .3660\)
\(c. \ Pr(K \geq 1) = 1 - Pr(K=0) = 1 - .3660 = .6340\)
\(d. \) Here \(n = 1600\), \(E(K) = 16\).
\(Pr(K = 16) = 1600 \binom{16}{.01}^{16} (0.99)^{1584} = .0997\)

5.15  
Here \(\pi = 1/2\), \(n = 4\), \(K = \) number of boys.
\(Pr(K > 2) = Pr(K = 3) + Pr(K = 4) = 4/16 + 1/16 = 5/16\)

5.17  
\(a. \ \mu = n \pi = (25)(.50) = 12.5,\)
\(\sigma = \sqrt{\pi(1-\pi)} = \sqrt{25(.5)(1-.5)} = 2.5\)
\(b. \ \mu = n \pi = (25)(.50) = 12.5,\)
\(\sigma = \sqrt{(500-25)/(499)} \sqrt{25(.5)(1-.5)} = 2.4391\)

5.19  
\(a. \) Here \(N = 52\), \(N_1 = 13\), \(N_0 = 39\), \(n = 13\), and \(k = 7\).
\(f(7) = \frac{13 \binom{7}{.01}^7 \binom{39}{6}}{52 \binom{13}{3}} = 0.0088\)
\(b. \ \mu = n \pi = 13(1/4) = 13/4\)
\(\sigma = \sqrt{\frac{N-n}{N-1}} \sqrt{n \cdot \pi(1-\pi)} = \sqrt{39/51} \sqrt{13(1/4)(3/4)} = 1.3653\)
\(c. \ Pr(7 \text{ spades or 7 clubs or 7 diamonds or 7 hearts}) = 4(0.0088) = 0.0353\)

5.21  
Let \(K\) be the number that recover. Here \(\pi = 1 - 63/877 = .9282\), \(n = 10\).
\(a. \ Pr(K = 10) = (0.9282)^{10} = 0.4745\)
\(b. \ Pr(K = 8) = 0.1279\)
\(c. \ Pr(K \geq 8) = 0.9697\)
5.23  a.

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b. See histogram in Answers section of book.

5.25  Let K be the number of girls.
Pr(3–2 split) = Pr(K=2) + Pr(K = 3) = 5/16 + 5/16 = 10/16 = 0.6250

5.27  Hypergeometric. Here N = 20, N_1 = 4, N_0 = 16, n = 5;

\[ f(k) = \frac{\binom{4}{k} \binom{16}{5-k}}{\binom{20}{5}} \]

a. \[ f(0) = \frac{\binom{4}{0} \binom{16}{5}}{\binom{20}{5}} = 0.2817 \]

b. \[ f(4) = \frac{\binom{4}{4} \binom{16}{1}}{\binom{20}{5}} = 0.0010 \]

c. \[ f(1) = \frac{\binom{4}{1} \binom{16}{4}}{\binom{20}{5}} = 0.4696 \]

d. \[ 1 - f(0) = 1 - 0.2817 = 0.7183 \]

e. \[ \mu = n\pi = 5(4/20) = 1 \]

f. \[ \sigma = \sqrt{\frac{20-5}{20-1} \cdot \frac{5(4/20)(16/20)}{4}} = 0.7947 \]

5.29  Hypergeometric. Here N = 16, N_1 = 5, N_0 = 11, n = 3.

\[ f(k) = \text{Pr}(k \text{ have worn tires}) = \frac{\binom{5}{k} \binom{11}{5-k}}{\binom{16}{3}} \]

Pr(0 have worn tires) = \[ \frac{\binom{5}{0} \binom{11}{3}}{\binom{16}{3}} = 0.2946 \]

Pr(At least one has worn tires) = \[ 1 - 0.2946 = 0.7054 \]
5.31 \( \pi = 1/48; \Pr(K \geq 1) = 1 - \Pr(K = 0) = 1 - \left( \frac{47}{48} \right)^2 \approx 0.2232 \)

5.33 a. Using hypergeometric. Here \( N = 120, N_1 = 80, N_0 = 40, n = 5. \)

\[
f(k) = \Pr(k \text{ have access}) = \frac{\binom{80}{k} \cdot \binom{40}{5-k}}{\binom{120}{5}}.
\]

\[
f(2) = \Pr(2 \text{ have access}) = \frac{\binom{80}{2} \cdot \binom{40}{3}}{\binom{120}{5}} \approx 0.1638
\]

b. Using binomial. Here \( n = 5, \pi = 80/120 = 2/3, k = 2 \)

\[
f(k) = \binom{5}{k} \left( \frac{2}{3} \right)^k \left( \frac{1}{3} \right)^{5-k}
\]

\[
f(2) = \binom{5}{2} \left( \frac{2}{3} \right)^2 \left( \frac{1}{3} \right)^3 \approx 0.1648
\]

c. No. The sample size \( n = 5 \) is less than 5% of population size \( N = 120. \) A population is considered small only if the sample is more than 5% of the population.