

# Chapter 5

## Random Variables for Success/Failure Experiments

**5.1**  $\mu = \pi = 1/13 = 0.0769,$   
 $\sigma = \sqrt{\pi(1-\pi)} = \sqrt{(1/13)(12/13)} = 0.2665$

**5.3**  $\mu = \pi = 6022/8023 = 0.7506,$   
 $\sigma = \sqrt{\pi(1-\pi)} = \sqrt{(6022/8023)(2001/8023)} = 0.4327$

**5.9** In general  $\mu = n\pi$  and  $\sigma = \sqrt{n\pi(1-\pi)}$

**a.** For  $n = 100$  and  $\pi = 1/3$ ,  $\mu = n\pi = 100/3 = 33.3333$  and  
 $\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{100(1/3)(2/3)} = \sqrt{200}/3 = 10\sqrt{2}/3 = 4.7140$

**b.** For  $n = 900$  and  $\pi = 1/3$ ,  $\mu = n\pi = 900/3 = 300$  and  
 $\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{900(1/3)(2/3)} = \sqrt{1800}/3 = 30\sqrt{2}/3 = 14.1421$

**c.** For  $n = 100$  and  $\pi = 2/3$ ,  $\mu = n\pi = 200/3 = 66.6667$  and  
 $\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{100(1/3)(2/3)} = \sqrt{200}/3 = 10\sqrt{2}/3 = 4.7140$

**d.** For  $n = 900$  and  $\pi = 2/3$ ,  $\mu = n\pi = 1800/3 = 600$  and  
 $\sigma = \sqrt{n\pi(1-\pi)} = \sqrt{900(1/3)(2/3)} = \sqrt{1800}/3 = 30\sqrt{2}/3 = 14.1421$

**5.11**  $\Pr(K = k) = {}_{10}C_k (.10)^k (.90)^{(10-k)}$

**a.**  $\Pr(K = 0) = {}_{10}C_0 (.10)^0 (.90)^{10} = (1)(1) (.90)^{10} = 0.3487$

**b.**  $\Pr(K = 1) = {}_{10}C_1 (.10)^1 (.90)^9 = (10)(.1) (.90)^9 = 0.3874$

**c.**  $\Pr(K = 2) = {}_{10}C_2 (.10)^2 (.90)^8 = (45)(.1)^2 (.90)^8 = 0.1937$

**d.**  $\Pr(K < 2) = \Pr(K = 0) + \Pr(K = 1) = .3487 + .3874 = 0.7361$

**e.**  $\Pr(K \geq 2) = 1 - \Pr(K < 2) = 1 - 0.7361 = 0.2639$

**f.**  $\Pr(K > 2) = 1 - \Pr(K \leq 2) = 1 - \{\Pr(K=0) + \Pr(K = 1) + \Pr(K = 2)\} = 1 - 0.9298 = 0.0702$

**5.13 a.**  $\Pr(K = 1) = {}_{100}C_1(.01)^1(.99)^{99} = .3697$

**b.**  $\Pr(K = 0) = {}_{100}C_0(.01)^0(.99)^{100} = .3660$

**c.**  $\Pr(K \geq 1) = 1 - \Pr(K = 0) = 1 - .3660 = .6340$

**d.** Here  $n = 1600$ ,  $E(K) = 16$ .

$\Pr(K = 16) = {}_{1600}C_{16}(.01)^{16}(.99)^{1584} = .0997$

**5.15** Here  $\pi = 1/2$ ,  $n = 4$ ,  $K =$  number of boys.

$\Pr(K > 2) = \Pr(K = 3) + \Pr(K = 4) = 4/16 + 1/16 = 5/16$

**5.17 a.**  $\mu = n\pi = (25)(.50) = 12.5$ ,

$\sigma = \sqrt{\pi(1-\pi)} = \sqrt{25(.5)(1-.5)} = 2.5$

**b.**  $\mu = n\pi = (25)(.50) = 12.5$ ,

$\sigma = \sqrt{(500-25)/(499)} \sqrt{25(.5)(1-.5)} = 2.4391$

**5.19 a.** Here  $N = 52$ ,  $N_1 = 13$ ,  $N_0 = 39$ ,  $n = 13$ , and  $k = 7$ .

$f(7) = \frac{{}_{13}C_7 \cdot {}_{39}C_6}{{}_{52}C_{13}} = 0.0088$

**b.**  $\mu = n\pi = 13(1/4) = 13/4$

$\sigma = \sqrt{\frac{N-n}{N-1}} \sqrt{n \cdot \pi(1-\pi)} = \sqrt{39/51} \sqrt{13(1/4)(3/4)} = 1.3653$

**c.**  $\Pr(7 \text{ spades or } 7 \text{ clubs or } 7 \text{ diamonds or } 7 \text{ hearts}) = 4(0.0088) = 0.0353$

**5.21** Let  $K$  be the number that recover. Here  $\pi = 1 - 63/877 = .9282$ ,  $n = 10$ .

**a.**  $\Pr(K = 10) = (.9282)^{10} = 0.4745$

**b.**  $\Pr(K = 8) = 0.1279$

**c.**  $\Pr(K \geq 8) = 0.9697$

5.23 a.

x	f(x)
0	0.1296
1	0.3456
2	0.3456
3	0.1536
4	0.0256

b. See histogram in Answers section of book.

5.25 Let K be the number of girls.  
 $\Pr(3-2 \text{ split}) = \Pr(K=2) + \Pr(K=3) = 5/16 + 5/16 = 10/16 = 0.6250$

5.27 Hypergeometric. Here  $N = 20$ ,  $N_1 = 4$ ,  $N_0 = 16$ ,  $n = 5$ ;

$$f(k) = \frac{{}^4C_k \cdot {}^{16}C_{5-k}}{{}^{20}C_5}.$$

a.  $f(0) = \frac{{}^4C_0 \cdot {}^{16}C_5}{{}^{20}C_5} = 0.2817$

b.  $f(4) = \frac{{}^4C_4 \cdot {}^{16}C_1}{{}^{20}C_5} = 0.0010$

c.  $f(1) = \frac{{}^4C_1 \cdot {}^{16}C_4}{{}^{20}C_5} = 0.4696$

d.  $1 - f(0) = 1 - 0.2817 = 0.7183$

e.  $\mu = n\pi = 5(4/20) = 1$

f.  $\sigma = \sqrt{\frac{20-5}{20-1}} \sqrt{5 \cdot (4/20)(16/20)} = 0.7947$

5.29 Hypergeometric. Here  $N = 16$ ,  $N_1 = 5$ ,  $N_0 = 11$ ,  $n = 3$ .

$$f(k) = \Pr(k \text{ have worn tires}) = \frac{{}^5C_k \cdot {}^{11}C_{3-k}}{{}^{16}C_3}.$$

$$\Pr(0 \text{ have worn tires}) = \frac{{}^5C_0 \cdot {}^{11}C_3}{{}^{16}C_3} = .2946$$

$$\Pr(\text{At least one has worn tires}) = 1 - .2946 = .7054$$

**5.31**  $\pi = 1/48$ ;  $\Pr(K \geq 1) = 1 - \Pr(K = 0) = 1 - (47/48)^{12} = 0.2232$

**5.33 a.** Using hypergeometric. Here  $N = 120$ ,  $N_1 = 80$ ,  $N_0 = 40$ ,  $n = 5$ .

$$f(k) = \Pr(k \text{ have access}) = \frac{80 C_k \cdot 40 C_{5-k}}{120 C_5}.$$

$$f(2) = \Pr(2 \text{ have access}) = \frac{80 C_2 \cdot 40 C_3}{120 C_5} = 0.1638$$

**b.** Using binomial. Here  $n = 5$ ,  $\pi = 80/120 = 2/3$ ,  $k = 2$

$$f(k) = {}_5C_k (2/3)^k (1/3)^{5-k}$$

$$f(2) = {}_5C_2 (2/3)^2 (1/3)^3 = 0.1648$$

**c.** No. The sample size  $n = 5$  is less than 5% of population size  $N = 120$ . A population is considered small only if the sample is more than 5% of the population.