Chapter 4
Discrete Random Variables

4.1 U and X are discrete random variables.

4.3 $F(x) = 0$ if $x < 0$
$F(x) = 27/64$ if $0 <= x < 1$
$F(x) = 54/64$ if $1 <= x < 2$
$F(x) = 63/64$ if $2 <= x < 3$
$F(x) = 1$ if $3 <= x$

4.5 $f(0) = Pr(X = 0) = .75$,
$f(1) = Pr(X = 1) = .25$,
$f(x) = 0$ elsewhere.

4.9 $\mu = \sum x \cdot f(x) = \{(0)(27/64) + (1)(27/64) + (2)(9/64) + (3)(1/64) =$
$\{0/64 + 27/64 + 18/64 + 3/64\} = 48/64 = 3/4$

$\sigma^2 = \sum (x-\mu)^2 \cdot f(x) = \{ (-3/4)^2 (27/64) + (1/4)^2 (27/64) + (5/4)^2 (9/64) + (9/4)^2 (1/64)\} =$
$36/64 = 9/16$

Or you can use the simpler formula
$\sigma^2 = E(X^2) - \mu^2 = (72/64) - (3/4)^2 = 9/16$
$\sigma = \sqrt{9/16} = 3/4$

4.11 $E(X) = \mu_X = \sum x \cdot f(x) = \{(0)(.50)^3 + (1)(3(.50)(.50)^2) + (2)(3(.50)^2(.50)) + (3)(.50)^3\} =$
1.50

$E(Y) = \mu_Y = \sum y \cdot f(y) = \{(0)(.50)^3 + (1)(3(.50)(.50)^2) + (2)(3(.50)^2(.50)) + (3)(.50)^3\} =$
1.50

$\sigma^2(X) = E(X^2) - \mu_X{}^2 = (3.0) - (1.50)^2 = .75$

$\sigma^2(Y) = E(Y^2) - \mu_Y{}^2 = (3.0) - (1.50)^2 = .75$

Note that $X + Y = 3$ for each possible outcome. So $E(X + Y) = 3$ and $SD(X + Y) = 0$.

4.13 $E(X) = \mu_X = \sum x \cdot f(x) = \{(0)(.48)^3 + (1)(3(.52)(.48)^2) + (2)(3(.52)^2(.48)) + (3)(.52)^3\} =$ 1.56

$E(Y) = \mu_Y = \sum y \cdot f(y) = \{(0)(.52)^3 + (1)(3(.48)(.52)^2) + (2)(3(.48)^2(.52)) + (3)(.48)^3\} = 1.44$

$\sigma^2(X) = E(X^2) - \mu_X^2 = (3.1824) - (1.56)^2 = .7488, \sigma = 0.8653$

$\sigma^2(Y) = E(Y^2) - \mu_Y^2 = (2.8224) - (1.44)^2 = .7488, \sigma = 0.8653$

Note that X + Y = 3, for each possible outcome. So E(X + Y) = 3 and SD(X + Y) = 0.

4.15 $\mu = \sum_{x=0}^{3} xf(x) = (0)(1/8) + (1)(3/8) + (2)(3/8) + (3)(1/8) = 12/8 = 3/2$.

$\sigma = \sqrt{E(X^2) - \mu^2} =$
$\sqrt{(0)^2(1/8) + (1)^2(3/8) + (2)^2(3/8) + (3)^2(1/8) - (3/2)^2} = \sqrt{24/8 - 9/4} = \sqrt{3}/2$

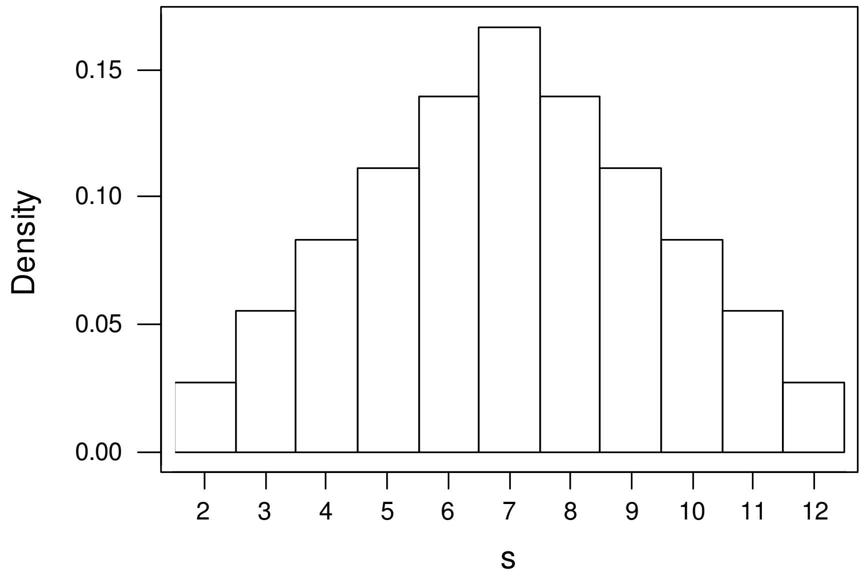
4.17 $\mu = \frac{n+1}{2} = \frac{11}{2} = 5.5$ $\quad\sigma = \sqrt{\frac{n^2-1}{12}} = \frac{\sqrt{33}}{2} = 2.8723$

4.19 $\mu = \{0(1/2) + 1(1/2)\} = 1/2,\ \sigma^2 = E(U^2) - E^2(U) = 1/2 - 1/4 = 1/2,\ \sigma = 0.707$

4.21 a.

x	f(x)	xf(x)	$x^2f(x)$
2	1/36	2/36	4/36
3	2/36	6/36	18/36
4	3/36	12/36	48/36
5	4/36	20/36	100/36
6	5/36	30/36	180/36
7	6/36	42/36	294/36
8	5/36	40/36	320/36
9	4/36	36/36	324/36
10	3/36	30/36	300/36
11	2/36	22/36	242/36
12	1/36	12/36	144/36
Sum	36/36 = 1	μ = 252/36 = 7	1974/36 = 329/6

b.



c. $\mu_S = \frac{252}{36} = 7$

d. $\sigma_S = \sqrt{\frac{329}{6} - 7^2} = \sqrt{\frac{35}{6}} = \frac{\sqrt{210}}{6} = 2.4152$

4.23 **a.**

k	f(k)
0	30/56
1	24/56
2	2/56

b. F(k) = 0 if k < 0
F(k) = 30/56 if 0 ≤ k < 1
F(k) = 54/56 if 1 ≤ k < 2
F(k) = 56/56 = 1 if 2 ≤ k

Sketch omitted. The graph is an increasing step function that has jumps of 30/56, 24/56, and 2/56 at 0, 1, and 2, respectively.

c. $\mu = (0)(30/56) + (1)(24/56) + (2)(2/56) = 26/36 = 1/2$

d. $\sigma = \sqrt{\frac{32}{56} - (\frac{1}{2})^2} = \sqrt{\frac{9}{28}} = \frac{3\sqrt{7}}{14} = .5669$

4.25 f(0) = F(0) – F(–1) = 1/8,
f(1) = F(1) – F(0) = 3/8,
f(2) = F(2) – F(1) = 3/8,
f(3) = F(3) – F(2) = 1/8,
f(x) = 0 elsewhere.

4.27 **a.**

x	f(x)	xf(x)	x^2f(x)
0	$(.95)^4 = 0.8145$	0	0
1	$4(.05)(.95)^3 = 0.1715$	0.1715	0.1715
2	$6(.05)^2(.95)^2 = 0.0135$	0.0271	0.0542
3	$4(.05)^3(.95) = 0.0005$	0.0014	0.0043
4	$(.05)^4 = 0.0000$	0.0000	0.0001
Sum	1	$\mu = 0.2$	0.23

b. $\mu = 0.2$

c. $\sigma^2 = .23 - .2^2 = .19$, $\sigma = \sqrt{.19} = .4359$

4.29 **a**. f(0) = (35/40)(34/39),
f(1) = 2(5/40)(35/39),
f(2) = (5/40)(4/39),
f(x) = 0 otherwise

b. f(0) = $(.96)^2$,
f(1) = 2(0.96)(0.04),
f(2) = $(0.04)^2$,
f(x) = 0 otherwise.