Chapter 4
Discrete Random Variables

4.1 U and X are discrete random variables.

4.3
\[
F(x) = \begin{cases} 
0 & \text{if } x < 0 \\
27/64 & \text{if } 0 \leq x < 1 \\
54/64 & \text{if } 1 \leq x < 2 \\
63/64 & \text{if } 2 \leq x < 3 \\
1 & \text{if } 3 \leq x 
\end{cases}
\]

4.5
\[
f(0) = Pr(X = 0) = 0.75, \\
f(1) = Pr(X = 1) = 0.25, \\
f(x) = 0 \text{ elsewhere.}
\]

4.9
\[
\mu = \sum x \cdot f(x) = \{(0)(27/64) + (1)(27/64) + (2)(9/64) + (3)(1/64) = 0/64 + 27/64 + 18/64 + 3/64 \} = 48/64 = 3/4
\]
\[
\sigma^2 = \sum (x - \mu)^2 \cdot f(x) = \{(-3/4)^2(27/64) + (1/4)^2(27/64) + (5/4)^2(9/64) + (9/4)^2(1/64)\} = 36/64 = 9/16
\]
Or you can use the simpler formula
\[
\sigma^2 = E(X^2) - \mu^2 = (72/64) - (3/4)^2 = 9/16
\]
\[
\sigma = \sqrt{9/16} = 3/4
\]

4.11
\[
E(X) = \mu_X = \sum x \cdot f(x) = \{(0)(.50)^3 + (1)(.50)(.50)^2 + (2)(.50)^2(.50) + (3)(.50)^3 \} = 1.50
\]
\[
E(Y) = \mu_Y = \sum y \cdot f(y) = \{(0)(.50)^3 + (1)(.50)(.50)^2 + (2)(.50)^2(.50) + (3)(.50)^3 \} = 1.50
\]
\[
\sigma^2(X) = E(X^2) - \mu_X^2 = (3.0) - (1.50)^2 = .75
\]
\[
\sigma^2(Y) = E(Y^2) - \mu_Y^2 = (3.0) - (1.50)^2 = .75
\]

Note that X + Y = 3 for each possible outcome. So E(X + Y) = 3 and SD(X + Y) = 0.
4.13 \[ E(X) = \sum x \cdot f(x) = \{(0)(.48)^3 + (1)(3(.52)(.48)^2) + (2)(3(.52)^2)(.48)) + (3)(.52)^3\} = 1.56 \]

\[ E(Y) = \sum y \cdot f(y) = \{(0)(.52)^3 + (1)(3(.48)(.52)^2) + (2)(3(.48)^2)(.52)) + (3)(.48)^3\} = 1.44 \]

\[ \sigma^2(X) = E(X^2) - \mu^2 = (3.1824) - (1.56)^2 = .7488, \sigma = 0.8653 \]

\[ \sigma^2(Y) = E(Y^2) - \mu_Y^2 = (2.8224) - (1.44)^2 = .7488, \sigma = 0.8653 \]

Note that X + Y = 3, for each possible outcome. So E(X + Y) = 3 and SD(X + Y) = 0.

4.15 \[ \mu = \sum_{x=0}^{3} x \cdot f(x) = (0)(1/8) + (1)(3/8) + (2)(3/8) + (3)(1/8) = 12/8 = 3/2 \ . \]

\[ \sigma = \sqrt{E(X^2) - \mu^2} = \sqrt{(0)^2(1/8) + (1)^2(3/8) + (2)^2(3/8) + (3)^2(1/8) - (3/2)^2} = \sqrt{24/8 - 9/4} = \sqrt{3}/2 \]

4.17 \[ \mu = \frac{n+1}{2} = \frac{11}{2} = 5.5 \quad \text{and} \quad \sigma = \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{33}{2}} = 2.8723 \]

4.19 \[ \mu = \{0(1/2) + 1(1/2)\} = 1/2, \quad \sigma^2 = E(U^2) - E^2(U) = 1/2 - 1/4 = 1/2, \quad \sigma = 0.707 \]

4.21

<table>
<thead>
<tr>
<th>(x)</th>
<th>f(x)</th>
<th>xf(x)</th>
<th>x^2f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1/36</td>
<td>2/36</td>
<td>4/36</td>
</tr>
<tr>
<td>3</td>
<td>2/36</td>
<td>6/36</td>
<td>18/36</td>
</tr>
<tr>
<td>4</td>
<td>3/36</td>
<td>12/36</td>
<td>48/36</td>
</tr>
<tr>
<td>5</td>
<td>4/36</td>
<td>20/36</td>
<td>100/36</td>
</tr>
<tr>
<td>6</td>
<td>5/36</td>
<td>30/36</td>
<td>180/36</td>
</tr>
<tr>
<td>7</td>
<td>6/36</td>
<td>42/36</td>
<td>294/36</td>
</tr>
<tr>
<td>8</td>
<td>5/36</td>
<td>40/36</td>
<td>320/36</td>
</tr>
<tr>
<td>9</td>
<td>4/36</td>
<td>36/36</td>
<td>324/36</td>
</tr>
<tr>
<td>10</td>
<td>3/36</td>
<td>30/36</td>
<td>300/36</td>
</tr>
<tr>
<td>11</td>
<td>2/36</td>
<td>22/36</td>
<td>242/36</td>
</tr>
<tr>
<td>12</td>
<td>1/36</td>
<td>12/36</td>
<td>144/36</td>
</tr>
<tr>
<td>Sum</td>
<td>36/36</td>
<td>(\mu = 252/36 = 7)</td>
<td>1974/36 = 329/6</td>
</tr>
</tbody>
</table>
b. 

![Histogram of density vs. s]


c. \[ \mu_s = \frac{252}{36} = 7 \]

d. \[ \sigma_s = \sqrt{\frac{329}{6} - 7^2} = \sqrt{\frac{35}{6}} = \sqrt{\frac{210}{6}} = 2.4152 \]
4.23  

a. 

<table>
<thead>
<tr>
<th>k</th>
<th>f(k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30/56</td>
</tr>
<tr>
<td>1</td>
<td>24/56</td>
</tr>
<tr>
<td>2</td>
<td>2/56</td>
</tr>
</tbody>
</table>

b. 

\[ F(k) = \begin{cases} 
0 & \text{if } k < 0 \\
30/56 & \text{if } 0 \leq k < 1 \\
24/56 & \text{if } 1 \leq k < 2 \\
2/56 & \text{if } 2 \leq k 
\end{cases} \]

Sketch omitted. The graph is an increasing step function that has jumps of 30/56, 24/56, and 2/56 at 0, 1, and 2, respectively.

c. \[ \mu = (0)(30/56) + (1)(24/56) + (2)(2/56) = 26/36 = 1/2 \]

d. \[ \sigma = \sqrt{\frac{32}{56} - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{9}{28}} = \frac{3\sqrt{7}}{14} = .5669 \]

4.25  

f(0) = F(0) - F(-1) = 1/8,
f(1) = F(1) - F(0) = 3/8,
f(2) = F(2) - F(1) = 3/8,
f(3) = F(3) - F(2) = 1/8,
f(x) = 0 otherwise.

4.27  

a. 

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>xf(x)</th>
<th>x^2f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(.95)^2 = 0.8145</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4(.05)(.95)^2 = 0.1715</td>
<td>0.1715</td>
<td>0.1715</td>
</tr>
<tr>
<td>2</td>
<td>6(.05)^2(.95) = 0.0135</td>
<td>0.0271</td>
<td>0.0542</td>
</tr>
<tr>
<td>3</td>
<td>4(.05)^2(.95) = 0.0005</td>
<td>0.0014</td>
<td>0.0043</td>
</tr>
<tr>
<td>4</td>
<td>(.05)^2 = 0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Sum</td>
<td>1</td>
<td>\mu = 0.2</td>
<td>0.23</td>
</tr>
</tbody>
</table>

b. \[ \mu = 0.2 \]

c. \[ \sigma^2 = .23 - .2^2 = .19, \quad \sigma = \sqrt{.19} = .4359 \]

4.29  

a.  

f(0) = (35/40)(34/39),
f(1) = 2(5/40)(35/39),
f(2) = (5/40)(4/39),
f(x) = 0 otherwise

b.  

f(0) = (96)^2, 
f(1) = 2(0.96)(0.04), 
f(2) = (0.04)^2, 
f(x) = 0 otherwise.